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- A technical note -

by

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Abstract. This paper reconsiders Teunter's remanufacturing model. It provides the analytical form of the proposed model and gives the explicit solution. In contrast to Teunter, it is shown that the optimal (noninteger) manufacturing and remanufacturing batches might be both greater than one. Furthermore the optimal reuse rate is determined excplicitely.

Key words: Production, EOQ model, waste disposal, cost minimization, remanufacturing

1. Introduction

Quantitative models for inventory systems with remanufacturing provide an actual generalization of classical EOQ models. A number of authors have proposed such a models. Our papers deals with one of these proposals, we investigate the model of Teunter [13].

Teunter in his work has stated that in the proposed model there should be either no more than one manufacturing batch and no more than one remanufacturing batch in a cycle. A cycle is a sequence of activities with a fixed number of batches.

The goal of the paper is to reconsider the Teunter's model. First the explicit model will be discussed and a solution is given for this model, because the author has neglected to describe the explicit model. After solving the problem, we give a counterexample, where the manufacturing and remanufacturing batches are strictly greater than one. By this counterexample we show that the Teunter's graphical proof, that one of the batch numbers equal to one is not correct. In fact, he proved this property of the batch numbers for the assumption that only relatively prime (coprime) batch numbers of manufacturing and remanufacturing are considered, or in other words, for batch numbers with a greatest common divisors greater than one.

2. The model

Teunter has investigated in his model the following activities:

- remanufacturing,
- disposal and
- manufacturing.

Let a *cycle* be the above-mentioned schedule with fixed batch sizes for manufacturing and remanufacturing. In a planning period there is *only one* cycle. (This can be proved very easily by grouping the remanufacturing and manufacturing lots.)

The goal of the decision maker is to minimize the cost for manufacturing and remanufacturing batch numbers and sizes and for the reuse rate. There are EOQ-oriented setup and holding costs for remanufacturing and manufacturing, linear production and remanufacturing costs, linear disposal cost and holding cost for non-serviceable items.

The notations of the model are the following:

System parameters:

- T length of the planning horizon,
- r return rate ($0 \le r \le 1$),
- λ rate of demand.

Cost parameters:

- K_m setup cost for manufacturing,
- K_r setup cost for remanufacturing,
- h_m holding cost for manufacturing items,
- h_r holding cost for remanufacturing items,
- h_n holding cost for non-serviceable items,
- c_m manufacturing cost,
- c_r remanufacturing cost,
- c_d cost for disposing one non-serviceable item.

Decision variables:

- Q_m batch size for manufacturing,
- Q_r batch size for remanufacturing,
- M number of manufacturing batches,
- R number of remanufacturing batches,
- u reuse rate ($0 \le u \le r$).

We assume that all parameters and the decisions variables are nonnegative numbers. We will describe the mathematical model with some application.

First we show some relation.

1) Relation for serviceable stock:

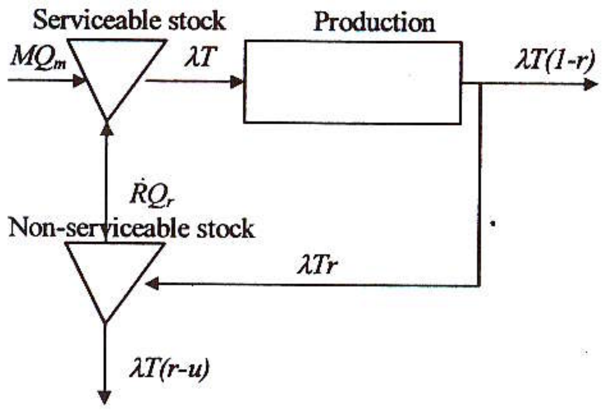
$$MQ_m + RQ_r = \lambda T \qquad (1)$$

2) Relation for non-serviceable stock:

$$RQ_r + \lambda T(r-u) = \lambda Tr \qquad (2)$$

$$0 \le u \le r$$

Fig. 1. Material flow in the model



The linear systems (1) and (2) can be solved for Q_r and Q_m:

$$Q_m = \frac{\lambda(1-u)T}{M} \qquad (3)$$

and

$$Q_r = \frac{\lambda uT}{R} \qquad (4)$$

In relation (4) it can be $u=0$, and $Q_r=0$. It means that all returned parts are disposed, there is no reuse in system and the management problem turns into a simple inventory problem. Another interesting case is, if the return rate equal to reuse rate ($u=r$). This case shows an example, when all returned parts are reused and there is no disposal activity. Identity (3) and (4) will be useful to create our cost function.

Now we describe the total cost function. This we make in two steps. First we investigate the cost function for serviceable and the non-serviceable parts.

Cost for serviceable items:

$$\left(RK_r + Rh_r \frac{Q_r^2}{2\lambda} + c_r RQ_r\right) + \left(MK_m + Mh_m \frac{Q_m^2}{2\lambda} + c_m MQ_m\right)$$

setup costs + holding costs + (re)manufacturing costs

It is easy to obtain using the EOQ-model.

Cost for non-serviceable items:

$$h_n \frac{Q_r^2}{2\lambda}\left\{R^2\left(\frac{1}{r}-1\right)+R\right\} + c_d \lambda T(r-u)$$

holding costs + disposal cost

To prove this relation, we introduce the maximum stock level for the non-serviceable parts I_n^{max}. We know that RQ_r unit will be remanufactured, after the Rth remanufacturing the inventory level is zero, and the returned parts could be described as $(R-1)\frac{Q_r}{\lambda}\lambda r$.

Then it is true the following:

$$I_n^{\max} - RQ_r + (R-1)\frac{Q_r}{\lambda}\lambda r = 0 \Rightarrow I_n^{\max} = RQ_r - (R-1)Q_r r = Q_r[R(1-r)+r]$$

The inventory holding costs for non-serviceable items consist of two parts: holding costs during the remanufacturing and holding costs during the manufacturing. First we construct the holding costs during the manufacturing. It is easy to obtain

$$I_n^{\max} \frac{I_n^{\max}}{2\lambda r} = \frac{Q_r^2}{2\lambda} \frac{[R(1-r)+r]^2}{r}$$

Let us now model the remanufacturing activities for returned parts. In the ith interval the initial inventory level is

$$I_i^b = I_n^{\max} - iQ_r + (i-1)rQ_r = Q_r[i(r-1) + R(1-r)]$$

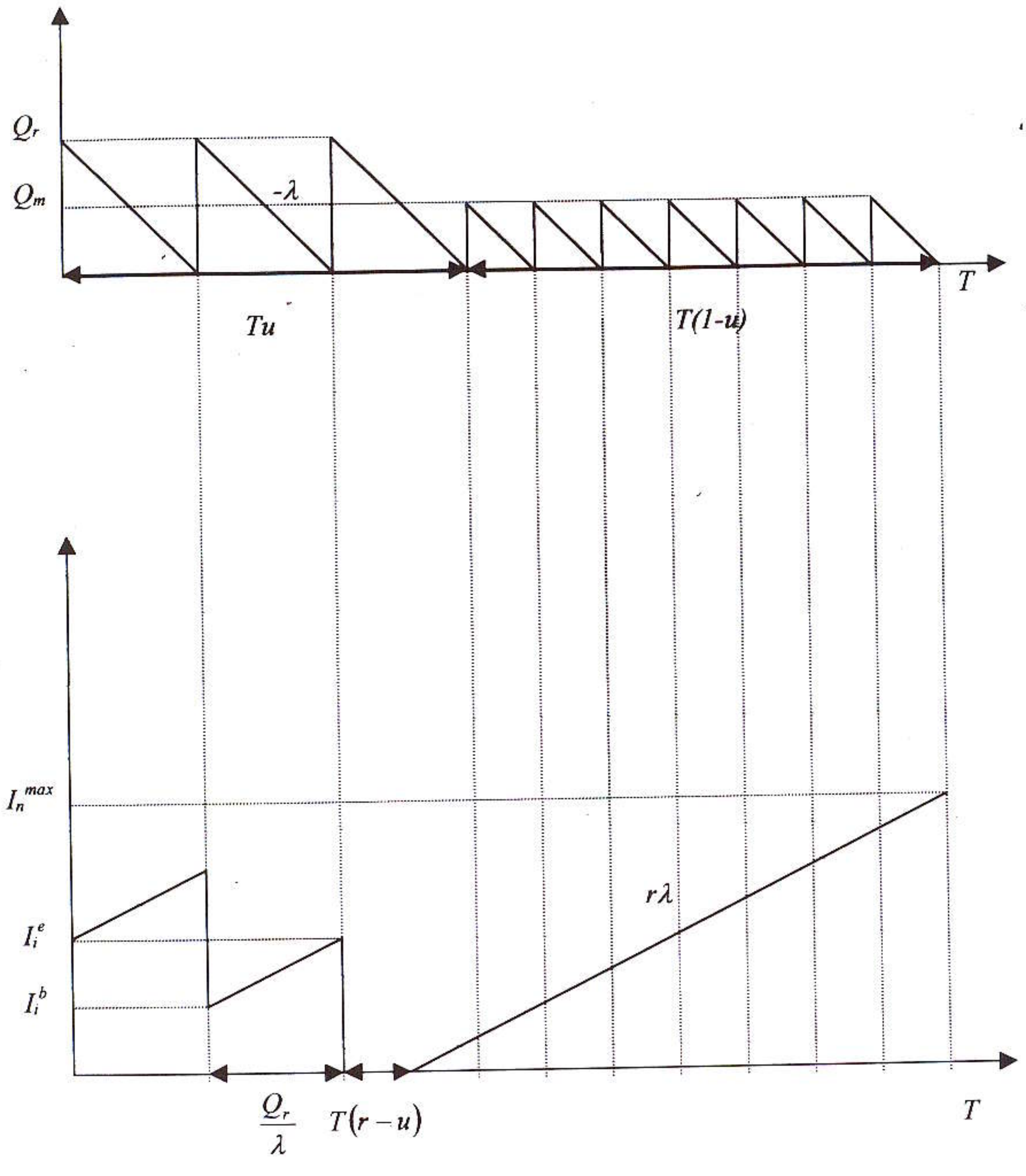
and the ending inventory level

$$I_i^e = I_n^{\max} - iQ_r + irQ_r = Q_r[i(r-1) + R(1-r) + r].$$

The holding costs during the remanufacturing are defined as follows

$$\frac{Q_r}{2\lambda}\sum_{i=1}^{R-1}\left(I_i^b + I_i^e\right).$$

Fig. 2. Modelling the inventory policy (R=3, M=7)



If we summarize for i

$$\sum_{i=1}^{R-1}\left(I_i^b + I_i^e\right) = \sum_{i=1}^{R-1}\left[\{i(r-1)+R(1-r)\}+\{i(r-1)+R(1-r)+r\}\right] =$$
$$= Q_r\left\{R^2(1-r)-R(2r-1)-r\right\}$$

and the holding costs for the non-serviceable items

$$h_n\left(\frac{Q_r}{2\lambda}\sum_{i=1}^{R-1}\left(I_i^b + I_i^e\right)+I_n^{\max}\frac{I_n^{\max}}{2\lambda r}\right) = h_n\frac{Q_r^2}{2\lambda}\left\{R^2\left(\frac{1}{r}-1\right)+R\right\}$$

so the total cost function $F(Q_r,Q_m,R,M,u)$ are known:

$$F(Q_r,Q_m,M,R,u) = \left(RK_r + Rh_r\frac{Q_r^2}{2\lambda}+c_rRQ_r\right)+\left(MK_m + Mh_m\frac{Q_m^2}{2\lambda}+c_mMQ_m\right)+$$
$$+h_n\frac{Q_r^2}{2\lambda}\left\{R^2\left(\frac{1}{r}-1\right)+R\right\}+c_d\lambda T(r-u)$$

We formulate the total cost substituting the relation (3) and (4) in the above-described cost functions

$$\widetilde{F}(M,R,u) = RK_r + R\frac{h_r}{2\lambda}\left(\frac{\lambda uT}{R}\right)^2 + c_m\lambda uT + MK_m + M\frac{h_m}{2\lambda}\left(\frac{\lambda(1-u)T}{M}\right)^2 + c_m\lambda(1-u)T$$
$$+\frac{h_n}{2\lambda}\left(\frac{\lambda uT}{R}\right)^2\left\{R^2\left(\frac{1}{r}-1\right)+R\right\}+c_d\lambda(r-u)T$$

Function $\widetilde{F}(R,M,u)$ can be written in a simpler form:

$$\widetilde{F}(R,M,u) = RK_r + \frac{h_r+h_n}{2}\frac{\lambda u^2T^2}{R}+K_mM+\frac{h_m}{2}\frac{\lambda(1-u)^2T^2}{M}+$$
$$+\frac{h_n}{2}\lambda u^2T^2\left(\frac{1}{r}-1\right)+\lambda uT\left(c_r-c_m-c_d\right)+\lambda T\left(c_m+c_dr\right)$$

This cost function is to minimize for values R, M and u.

3. Optimal solution of the model

The optimal solution will be constructed stepwise, the sequence of the optimization is the following:

- R, M and
- u.

From the convexity of function $\widetilde{F}(R,M,u)$ in R and M follows

$$R^o = uT\sqrt{\frac{\lambda(h_r + h_n)}{2K_r}}$$

$$M^o = (1-u)T\sqrt{\frac{\lambda h_m}{2K_m}}$$

and the simpler cost function after substituting R^o and M^o

$$\widetilde{F}1(u) = uT\sqrt{2\lambda K_r(h_r + h_n)} + (1-u)T\sqrt{2\lambda K_m h_m} + \frac{h_n}{2}\lambda u^2 T^2\left(\frac{1}{r}-1\right) + \\ + \lambda uT(c_r - c_m - c_d) + \lambda T(c_m + c_d r) =$$

$$u^2\lambda T^2\frac{h_n}{2}\left(\frac{1}{r}-1\right) + uT\left[\sqrt{2\lambda K_r(h_r + h_n)} - \sqrt{2\lambda K_m h_m} + \lambda(c_r - c_m - c_d)\right] + \\ \left\{T\sqrt{2\lambda K_m h_m} + \lambda T(c_m + c_d r)\right\}$$

Function $\widetilde{F}1(u)$ is quadratic and convex again in u, and u is between 0 and r.

$$u^o = \begin{cases} 0 & \widetilde{u} \le 0 \\ \widetilde{u} & \widetilde{u} \in (0,r) \\ r & \widetilde{u} \ge r \end{cases}$$

where

$$\widetilde{u} = \frac{\sqrt{2\lambda K_m h_m} - \sqrt{2\lambda K_r(h_r + h_n)} + \lambda(c_m + c_d - c_r)}{\lambda T\frac{h_n}{2}\left(\frac{1}{r}-1\right)}$$

With the help of variable u^o we can calculate variables Q_r^o and Q_m^o

$$Q_r^o = \frac{\lambda u^o T}{R^o} = \sqrt{\frac{2\lambda K_r}{h_r + h_n}}$$

and

$$Q_m^o = \frac{\lambda(1-u^o)T}{M^o} = \sqrt{\frac{2\lambda K_m}{h_m}}$$

So we have obtained the following

Theorem: The optimal solution for the reverse logistics problem is:

$$u^o = \begin{cases} 0 & \tilde{u} \le 0 \\ \tilde{u} & \tilde{u} \in (0, r) \\ r & \tilde{u} \ge r \end{cases},$$

$$\tilde{u} = \frac{\sqrt{2\lambda K_m h_m} - \sqrt{2\lambda K_r (h_r + h_n)} + \lambda(c_m + c_d - c_r)}{\lambda T \frac{h_n}{2}\left(\frac{1}{r} - 1\right)},$$

$$R^o = u^o T \sqrt{\frac{\lambda(h_r + h_n)}{2K_r}},$$

$$Q_r^o = \sqrt{\frac{2\lambda K_r}{h_r + h_n}},$$

$$M^o = (1-u^o)T\sqrt{\frac{\lambda h_m}{2K_m}},$$

$$Q_m^o = \sqrt{\frac{2\lambda K_m}{h_m}}.$$

The model is hereby completely solved.

4. A numerical example

To illustrate the results of the last section, we use the following dates:

System parameters:

$T = 20$,
$r = 0.8$,
$\lambda = 1$.

Cost parameters:

$K_m = 6.6\sqrt{2} \approx 9.334$,
$K_r = 8$,
[illegible] $\frac{33}{40}$ [illegible]
$h_r = 0.8$,
$h_n = 0.2$,
$c_m = 0.9$,
$c_r = 0.2$,
$c_d = 0.8$.

With this parameters we have the following optimal solution:

Decision variables:

- $Q_m^o = 4$,
- $M^o = 3$,
- $Q_r^o = 4$,
- $R^o = 2$,
- $u^o = 0.8$.

That both manufacturing and remanufacturing batch numbers are greater than one, is caused by the high setup costs. This example shows, that batch sizes are coprimes.

5. Conclusion

In this paper we have solved a remanufacturing problem. It was shown that the optimal number of both manufacturing and remanufacturing sizes can be strictly greater one. The optimal manufacturing policy is either all returned items to dispose without remanufacturing or all returned items to remanufacture or to apply all both activities.

The solution of the model considers exclusively such cases when both batch numbers greater than zero, but not integer. A possible generalization would be the consideration of integer numbers of batches.

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