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Cyclical Price Fluctuations Caused by Information Inertia Evidence from the German Call-by-Call Telephone Market

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Abstract: The 2002 prices of suppliers in the German call-by-call telephone market are rather dispersed, out-of-phase (uncorrelated), and show systematic down-up movements. In 2004, these prices are less dispersed, more in-phase and show more upwards runs than down-ups. In both years, we clearly do not observe Edgeworth cycles where prices move in parallel (in-phase). We present a model with demand inertia, caused by incomplete information about prices, where (out-of-phase and in-phase) cycles as well as competitive equilibria and tacit collusion equilibria exist. The transition from the 2002 cycles in the German call-by-call market to more constant prices in 2004 may be due to parameter changes, such as customers possessing improved information.

Key words: Price cycles, incomplete information, telephone market
JEL: numbers: C 73, D 43

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I. Evidence of cyclical price fluctuations

Since 1998, the German telephone market has been liberalised. The incumbent Deutsche Telekom (DT) is no longer a monopolist but is faced with competition from some 30 other firms. Customers may switch to another provider or remain customer of DT (and pay the related fixed costs) but buy service on a call-by-call basis. In the latter case, a customer has to pre-dial the number of the provider she selects and then, in the case of national calls, she continues to dial the DT-number of the participant she wants to be connected with.

Thanks to the liberalisation, the price of telephone calls has decreased considerably – for domestic long distance calls, at the end of 2002 the average price on working days was only 7 % of that in 1997 (Regulierungsbeörde für Telekommunikation und Post, 2003). Such a development is undoubtedly a great success in terms of welfare, but it is not the topic of this paper. Instead of that we want to discuss a phenomenon we observed in the 2002 call-by-call market.

The suppliers in this market are free to change their prices as often as they want. Some of them (who provide also other services by telephone) are price constrained, but there are no restrictions on price *changes*. The usual sources of price information are either the internet or newspapers. Many local newspapers publish the lowest prices each Monday. So, usually you have a price list close to your telephone which has to be updated from time to time. If you call the cheapest supplier from a "fresh" price list you often get no connection – demand is above his capacity. If you call the cheapest supplier from an older price list you often make the experience that the price announced (not all suppliers announce the price in advance) is higher than the price in the list.

With this anecdotal experience in mind we had, in 2003, a closer look at the 2002 data. In Figure 1, the price development of the three suppliers with the lowest average prices (6.85 to 7.16 cent for a three minute call) are shown. First, apparently prices are highly volatile. Second, there is a *"systematic" up then down* of prices during the whole year. We observe similar price movements also for the next cheapest suppliers – up to No 13 who is the first with a nearly constant price (12 cent on average, only one price change). There were additional suppliers (DT required 36.7 cent!), but they probably did not play a significant role in this market segment.[2] The systematic character of the fluctuations can also be seen from Table 1. Those suppliers who change prices more than once apparently followed an up-down (or better down-up) pattern, possibly with constant prices for some periods but not with up or down movements consisting of several steps. The 3 + 34 = 37 price increases and the 31 + 8 = 39 price decreases are not distributed evenly. Chi-square tests show, in both cases a high level of significance ($p = 3 \bullet 10^{-7}$, $2 \bullet 10^{-4}$). It seemed that the suppliers followed a policy where, first, they attracted customers by low prices and, afterwards, exploited those customers who were not completely up to date with their price information.

		2002 A previous		2004 A previous	
		price increase	price decrease	price increase	price decrease
is followed by a	price increase	3	34	24	12
	price decrease	31	8	10	2

Table 1: The down and up of individual prices in 2002 and in the first eight month of 2004.

A third attribute of the price fluctuations in 2002 is the fact that prices did not move parallel to one another, i.e. they are not “in-phase”. This eye catching trait is supported by the correlation matrix of the nine cheapest suppliers. (See Table 2.) The correlation coefficients are, in general, rather low and half of them are negative (3 positive, 6 between –0.05 and +0.05, and 10 negative). There are two suppliers with identical prices. The missing values in the matrix are due to lacking parallel existence of the suppliers or to constant prices of one supplier during the period of co-existence.

[2] There is no information available on quantities sold.

The central question of this paper is whether such behavior could be equilibrium behavior. For this purpose we will investigate a symmetric two-person game with two possible prices. A customer knows with probability α the current prices or she knows only last period's prices. We assume the suppliers to use Markov strategies and to maximise "average values" over an infinite horizon. Requirements of symmetric equilibria and independence of initial conditions are used as selection devices. We indicate a region of prices and probabilities (for knowing the actual price) where equilibria with permanent price fluctuations exist. So, in this model price cycles *can* occur but need not occur.

In the German call-by-call market, there does not seem to be a necessity of price cycles either as a look at the data of 2004 shows (Figure 2). The cheapest supplier required a constant price and the down and up in 2002 seems to be substituted by upwards runs in 2004. The structure is still significantly different from a random distribution of the ups and downs (Chi-square tests: $p = 0.05$, 0.02) but it is apparently a different structure (Chi-square tests: $p = 2 \bullet 10^{-14}$, $3 \bullet 10^{-15}$). Prices *between suppliers*[3] do not fluctuate in 2004 (average σ^2 of the nine cheapest suppliers = 3.7) as much as in 2002 (average σ^2 of the nine cheapest suppliers = 10.7). Also the correlation matrix (Table 3) has a different structure: there are 14 positive correlation coefficients and 8 negative, 6 of them with the same supplier (none between –0.05 and +0.05). In addition, the distribution of the cheapest prices among the suppliers changed remarkably, even if we take into account that a supplier in 2002 existed only 45 % of the periods (average of the cheapest nine suppliers) while a supplier in 2004 existed 97 % of the periods.

In Section III, we discuss the generalization of our model with respect to more prices and more players. Our conclusion is that the cycles are equilibrial also under such conditions. In Section IV, we compare our model with some dynamic pricing models from the literature. Section V concludes.

[3] The variances of prices of single suppliers *between periods* are given in Tables 2 and 3. They do not vary considerably.

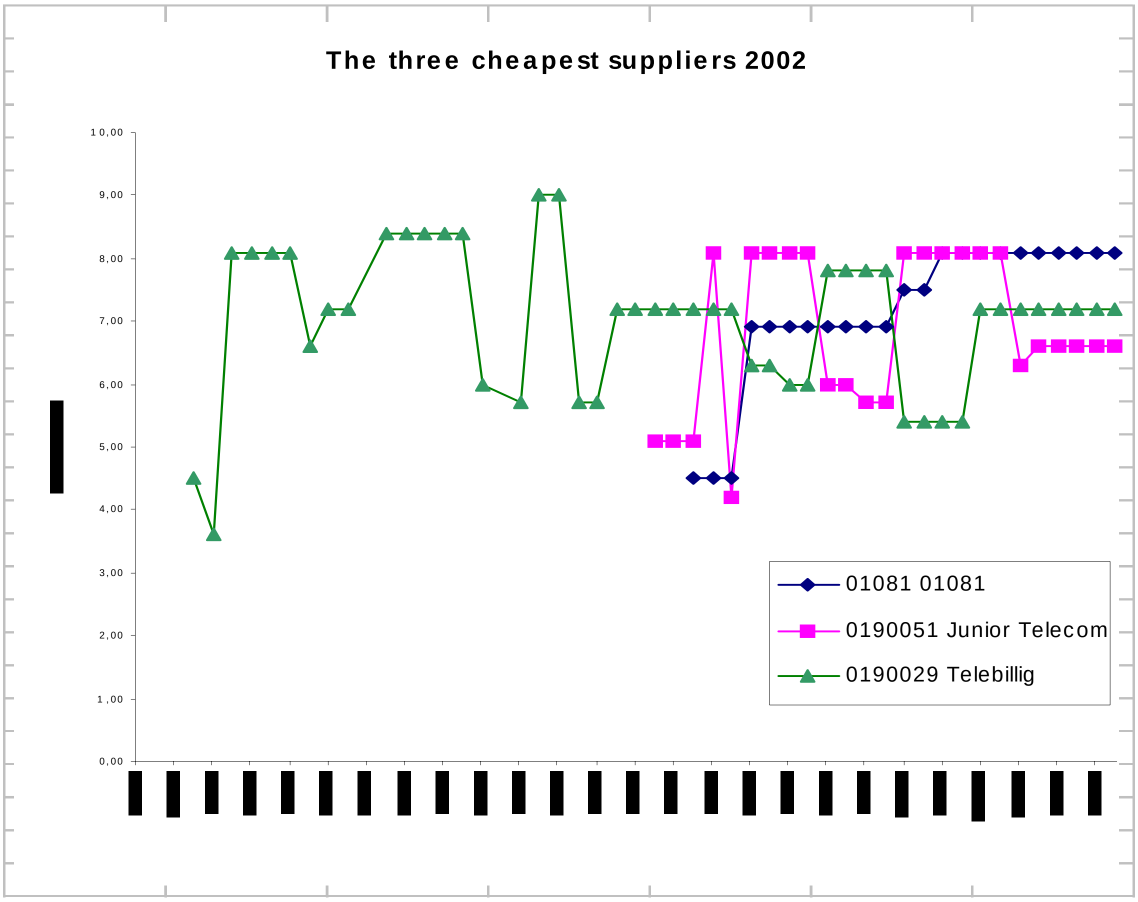

Figure 1: Price (three minute call) of the three cheapest suppliers in 2002.
Source: www.billiger-telefonieren.de (own graphical representation).

	2.	3.	4.	5.	6.	7.	8.	9.	mean price	Var	1*	2*	3*
1.Junior Tel.	0.4							0.1	6.9	1.6	26	10	12
2. 01081								0.6	7.2	1.4	4	30	0
3. Telebillig			1.0	-0.7		-0.2	-0.1	0	7.2	0.9	32	8	30
4. Pennyph.				-0.7		-0.2	-0.1	0	7.2	0.9	6	10	6
5. Ph.dump						-0.5	0	0	7.2	2.1	4	0	6
6. Ph.craft							-0.2	-0.6	7.3	0.6	4	26	6
7. Maxbell							-0.2	0	7.3	1.4	4	2	0
8. Fonfux								0	7.5	1.1	6	0	12
9. Telestunt									12.0	9.9	18	8	0
Others											*12*	*16*	*18*

Table 2: Correlation of prices of the 9 cheapest suppliers in 2002. * Frequencies (%) of cheapest (1), second cheapest (2), and third cheapest (3) price. In case of ties the lowest rank is used. Therefore the percentages need not add up to 100.

Figure 2: Price (three minute call) of the three cheapest suppliers in Jan. – Aug. 2004. Source: www.billiger-telefonieren.de (own graphical representation).

	2.	3.	4.	5.	6.	7.	8.	9.	mean price	Var	1*	2*	3*
1. Interroute									3.0	0	100	0	0
2. Freenet		0.3	-0.2	0.4	0.4	0.4		-0.6	5.0	0.4	0	48	24
3. 01081			0.7	0.7	0.5	0.8		-0.5	5.5	0.8	0	3	42
4. 01030				-0.1	0.7	0.7		-0.3	5.9	2.1	0	36	0
5. Callax					0.4	0.9		-0.7	6.8	5.6	0	3	30
6. Telestunt						1.0		-0.7	6.9	0.7	0	0	0
7. SurpriseT								-0.7	7.0	0.3	0	0	0
8. Telebillig									7.5	0	0	0	0
9. 01015									7.8	3.9	0	12	18
Others											*0*	*0*	*0*

Table 3: Correlation of prices of the 9 cheapest suppliers in 2004 (Jan. – August).
* Frequencies (%) of cheapest (1), second cheapest (2), and third cheapest (3) price. In case of ties the lowest rank is used. Therefore the percentages need not add up to 100.

II. A duopoly model

Let us assume that every consumer carries out an individual but fixed number of telephone calls per period, independent of the price of a call. The prices of the suppliers determine which supplier she chooses but they do not influence overall demand.

There are two suppliers with zero costs who determine their prices $p_i^t \in \{r,1\}$, i = 1, 2, 0 < r < 1, in every period t anew. With probability α, a consumer is informed about the prices of period t, with probability 1 - α, she only knows the prices of period t –1 (she has an outdated list). The consumer decides on the basis of the information she has, i.e. either according to the prices of period t or according to the prices of period t – 1. If $p_i^t < p_j^t$ (or $p_i^{t-1} < p_j^{t-1}$ if only the prices of period t – 1 are known) then i gets the whole demand. If prices are equal she decides by chance. The whole market demand is normalised to 1.

Thus there are four different price combinations as indicated in Table 4. The profits $U_1(K_{t-1}, K_t)$, K_{t-1}, $K_t \in$ {A, B, C, D} of firm (player) 1 are computed in Table 5. Please note that, in any case, consumers pay period t's prices. Take the case (K_{t-1}, K_t) = (D, A). With probability α, K_t is known and Firm 1 gets r/2. With probability 1 - α, only K_{t-1} is known which again implies that both firms get half the market demand, i.e. firm 1's profit is again $p_t \cdot 1/2 = r/2$.

		Firm 1's prices	
		r	1
Firm 2's prices	r	A	B
	1	C	D

Table 4: The possible price combinations.

		K_{t-1}	A	B	C	D
		Profit if t-1 is relevant				
K_t	p_t	Profit if t is relevant	$p_t/2$	0	p_t	$p_t/2$
A	r	r/2	r/2	αr/2	r(1-α/2)	r/2
B	1	0	(1 - α)/2	0	1 - α	(1 - α)/2
C	r	r	(1 + α)r/2	αr	r	(1 + α)r/2
D	1	1/2	1/2	α/2	1 - α/2	1/2

Table 5: Profits $U_1(K_{t-1}, K_t)$ of firm 1 in period t. A consumer knows K_t with probability α and she knows K_{t-1} (but not K_t) with probability 1 - α.

Concerning the long-term goal of the firms we assume, for the sake of simplicity, that the firms do not discount but want to maximise their "average pay-off", i.e.

$$(1) \qquad V_1 = \liminf_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} U(K_{t-1}, K_t)$$

with a certain initial K_0. In this objective function only those price combinations which occur infinitely often are relevant. So K_0 is only important because it may initialise a certain path $(K_t)_{t=0,1,\ldots}$ but not in itself. For example, if the price policies of the competitors imply a permanent fluctuation between price combinations B and D then $V_1 = \frac{1}{2}\,(U_1(B, D) + U_1(D, B))$.

By interchanging the roles of firm 1 and firm 2 we get the profits of firm 2.

Subgames in this infinite game are described by the price structure of the last period. In the following, we want to concentrate on Markov strategies, i.e. we require identical strategies in identical subgames. Thus a Markov strategy s_i of firm i is a mapping

(2) $s_i : \{A, B, C, D\} \rightarrow \{r, 1\}$.

So, we can describe s_i by

(3) $s_i = (x^i_A, x^i_B, x^i_C, x^i_D)$, $x^i_K \in \{r, 1\}$, K = A, B, C, D, i = 1, 2

Every pair of strategies results in a dynamic price structure as indicated by the examples in Figure 4. (a) is a case with permanent price fluctuations, evaluated $V_1 = \frac{1}{2}$ (U(B, C) + U(C, B)) by 1. (b) results in a permanent price structure D, evaluated U(D, D) by 1. (c) depends on the initial condition, the cycle is evaluated $\frac{1}{3}$ (U(A, B) + U(B, C) + U(C, A)). Permanent price structures as D in (b) and (c) are also called stationary points. They are special cycles.

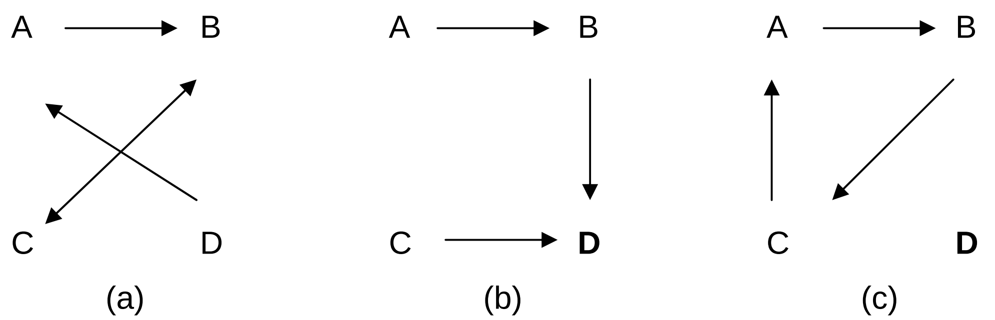

Figure 3: Price development under strategies (a): s_1 = (1, r, 1, r) and s_2 = (r, 1, r, r). (b): s_1 = (1, r, r, 1) and s_2 = (r, 1, r, 1). (c): s_1 = (r, r, r, 1) and s_2 = (r, 1, r, 1). Bold faces letters indicate stationary states.

The strategy profiles and the resulting dynamics as indicated in Figure 3 are isomorphic. So, we will often use the dynamics and the strategy profiles synonymously.

The number of Nash equilibria in our model is large. So we need selection principles.

Selection principles: (i) Only equilibria with deterministic strategies and symmetric cycles are selected.

(ii) Only equilibria which have "connected" dynamics are selected, i.e. those which have only one equilibrium cycle.

From (i) follows that only equilibria with cycles A, D, A $\leftrightarrow$ D, and B $\leftrightarrow$ C are selected. (ii) means that we will arrive at the same cycle from every possible state. In the following, we will investigate for which parameter constellations which of these cycles are supported by equilibria. The proofs of the following lemmas are in the appendix.

Lemma 1: The stationary point D is an equilibrium cycle if and only if

(4) $r \leq 2/3$.

Lemma 2: The cycle A $\leftrightarrow$ D is an equilibrium if and only if

(5) $r \leq 1/2$

(6) $\alpha \geq \frac{1}{4}$

(7) $\alpha \geq \frac{1-3r}{2-2r}$

Lemma 3: The cycle B $\leftrightarrow$ D is an equilibrium if and only if

(8) $\alpha \leq \frac{1-\frac{3}{2}r}{1-r}$

(9) $\alpha \leq \frac{1}{2}$.

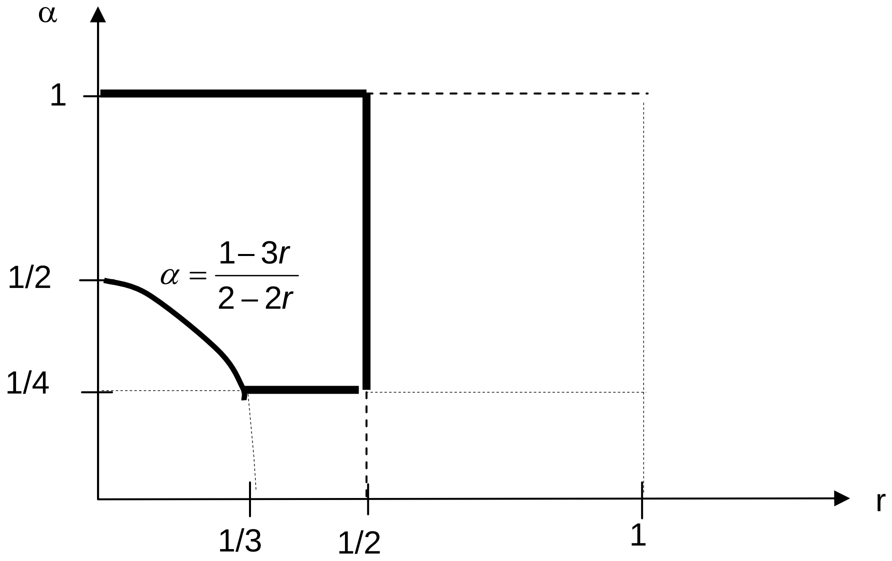

Figure 4: Parameter values which support the equilibrium cycle A ↔ D (Lemma 2).

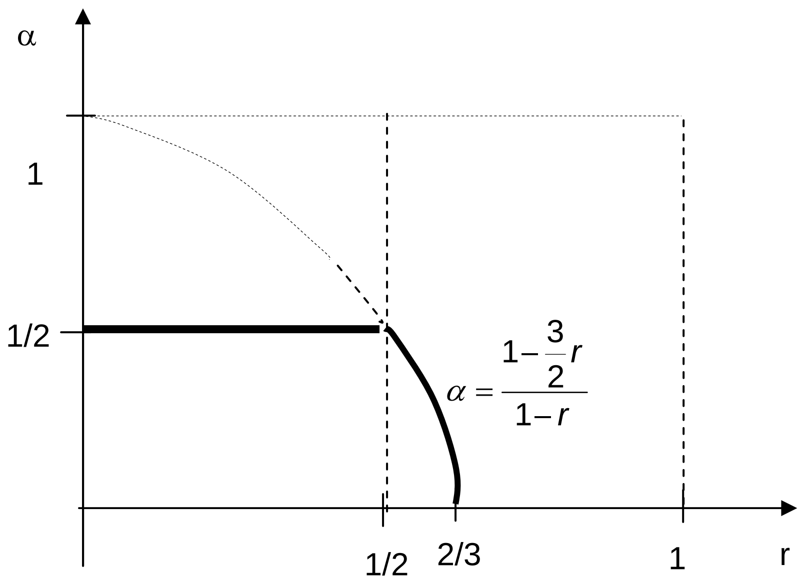

Figure 5: Parameter values which support the equilibrium cycle B ↔ C (Lemma 3).

Lemma 4: A is an equilibrium cycle if and only if

(10) $\alpha \geq (1-2r)/(1-r)$.

Theorem: The selected equilibrium cycles are those of Figure 6.

Proof: Lemmas 1, 2, 3, and 4.

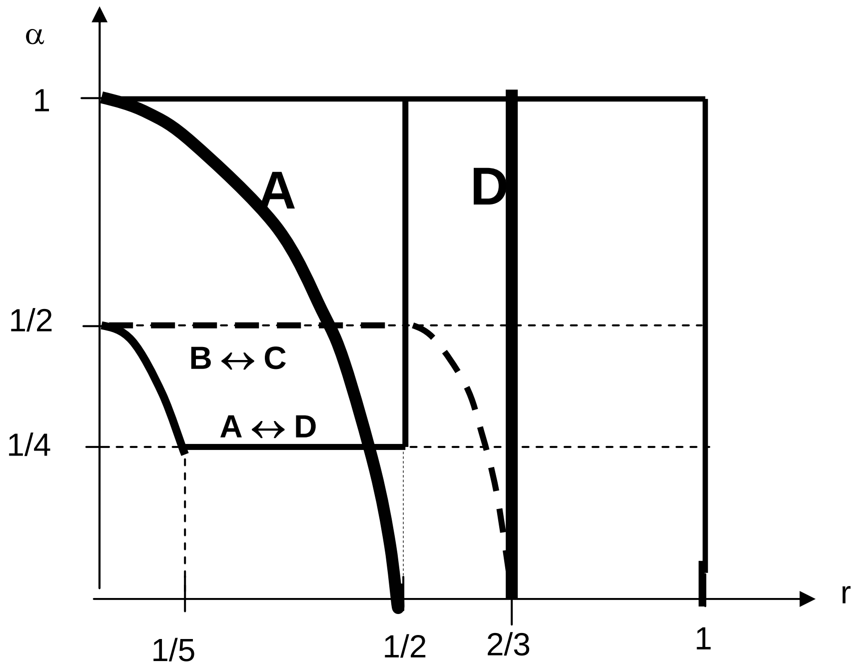

Figure 6: The selected equilibrium cycles: A is an equilibrium on the right of the A-curve, D on the left of the D-curve, B ↔ C below the dashed curve, and A ↔ D above the less bold line.

The competitive equilibrium cycle A occurs for large enough r and/or large enough α values, the tacit collusion equilibrium D with permanent high prices prevails for parameter values with small enough r. Cycles occur for low r values, the “out-of-phase” cycle B ↔ C for low α, and the “in-phase” cycle A ↔ D for large α.

In a model as simple as the duopoly model above we might ask why consumers do not anticipate the firms’ behavior. In a real market with several firms and “more stochastic” price fluctuations, it is difficult to develop “rational expectations” in the sense of perfect forecasts. Thus, aiming to describe the 2002 situation of the German call-by-call market with such a simple model, it seems to be sensible not to provide our consumers with rational expectations.

One can think of many other possibilities to “improve” the model, for example, introducing more prices or more suppliers or a more distributed price information. The first two extensions will be discussed in the next section. We will see that the main result, namely the *possibility* of "out-of-phase" price cycles, does not change.

III. More prices, more players

Maskin and Tirole (1988), in a duopoly model with a finite number of prices and alternating prices choices, show the existence of tacit collusion equilibria (corresponding to D) as well as “Edgeworth cycle”. (See Figure 7.)

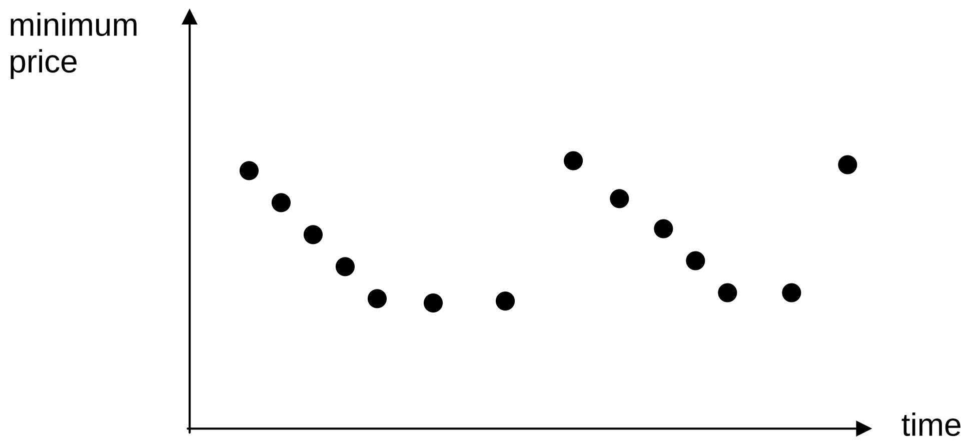

Figure 7: Edgeworth cycles

The competitors undercut one another until a “lowest price” is reached, then one of them jumps to a high price and the cycle starts anew. As this means “parallel” moves (correlated prices) such an equilibrium corresponds (if at all) to our cycle A $\leftrightarrow$ D, but not to B $\leftrightarrow$ C. Note that contrary to Maskin and Tirole (1988), D is not always an equilibrium and, therefore, not always selected by the principle of renegotiation proofness.

With n prices from the interval [r, 1] and with many prices close to r, a kind of trigger strategy is possible in our (generalised) model: If one player deviates from an equilibrium cycle the other player chooses one of these low prices one after the other until, finally, he chooses a price which allows the other to return to the cycle. (Note that he employs a Markov strategy.) The deviating player can choose always r and thus, with enough prices close to r, earn about r. Instead, he could choose alternating r and 1 and would get $(\alpha r + (1-\alpha))/2$. The latter profit, however, is always smaller than ½, the profit in D. Prices between r and 1 apparently do not make sense as an optimal reply against the described trigger strategy.

With m players, a joint trigger strategy would be even more effective. After one player deviates from the equilibrium path, all others, except 1, require r for many periods. After some time a price combination is reached where the deviating player is allowed to return to the cycle. The requirements of Markov strategies and of (ii) “one cycle” mean that the number of prices determine the maximal length of the “punishment periods”. Thus, the following lemma applies.

Lemma 5: With many prices, all equilibrium cycles which guarantee profits larger than max{r, (αr+1-α)}/(m-1) for all players are equilibria.

Corollary 1: With m players and “enough” prices, a sufficient condition for the tacit collusion equilibrium (1, 1, …, 1) to exist is

(11) $$r < (m-1)/m.$$

Are also Edgeworth Cycles equilibrial? If the price decreases “uniformly” from 1 to r then, with m players, every player earns

(12) $$\frac{1}{1-r}\int_r^1 \frac{x}{m}dx = \frac{1}{2m}(1+r)\,.$$

Corollary 2: With m players and “enough” prices, a sufficient condition for an Edgeworth cycle equilibrium to exist is r<(m-1)/(m+1).

The following plausibility argument emphasises the role of the price bounderies r and 1. If the number of players is large there is, in the face of the plethora of equilibria, a coordination problem. Players who doubt that trigger strategies are used are tempted, say in the case of D, to deviate by choosing a price “just below 1”. The same argument applies for all other stationary points. So let us assume that more complicate strategies result when the number of players is large. If a player, however, expects that, in all situations there is at least one other player requiring r’ or less, then it is always suboptimal for him to reply with prices larger than r’ and smaller than 1. A price between r’ and 1 would become relevant only if he had required the lowest price (or one of the lowest prices) in the round before – but then he is better off with an actual price of 1. If everybody chooses prices which are either =1 or $\leq$ r', then it seems to make sense never to use r', etc. In the end, we have a situation where everybody uses only prices r and 1. Such a situation would be self-stabilising if, in every situation, at least two suppliers chose r. The most simple of these cases are, for 3 players, the cycle (1,r,r)$\rightarrow$ (r,1,r) $\rightarrow$(r,r,1) $\rightarrow$(1,r,r) and, for 4 players, the cycle (1,1,r,r)$\leftrightarrow$(r,r,1,1). Also outside the cycle, the players must use strategies with "many r's".

This is no formal selection criterion but it emphasises the role of the extreme prices and the value of the previous section.

IV. Alternative Models

In a previous *simulation study* with discounting, with capacity restrictions, and with different selection criteria, we found already that price fluctuations could be equilibrial depending on the parameter values. (Baier and Bolle, 2004). Here, we have relied on exact poofs.

In the last section, we mentioned the model by Maskin and Tirole (1988) which focuses on Edgeworth Cycle Equilibria and Tacit Collusion Equlibria. There are apparently many other equilibria but they are not investigated.

Empirical studies have shown that, in the gasoline market, sometimes Edgeworth cycles are observed and sometimes constant prices (Eckert, 2002, 2003, Noel, 2003). In any case, our data do not show Edgeworth cycles, neither in 2002 nor in 2004. It also seems to be plausible that, in 2004, we observe prices closer to competitive prices than in 2002, i.e. we do not seem to have moved into the direction of Tacit Collusion.

While the driving force for Edgeworth cycles is the assumption of alternative moves, our cycles are based on incomplete information which causes *demand inertia*. Selten (1965), in his seminal article on competition with demand inertia mentions explicitly the possibility of lacking price information. The resulting prices in his model, however, do not show cycles but follow a "turnpike" movement. As far as we know, there are no other models of this kind which produce cycles. Rosenthal (1982) assumes another variant of demand inertia: In a duopoly, a buyer is loyal as long as “his” seller does not change his price. But this model does not lead to cycles either.

Green and Porter (1984) report stochastic price movements as consequences from stochastic demand and collusion. This might be an interesting model for other industries but, in our data, we do not find traces of complete collusion.

So it seems that this special form of demand inertia which we hypothesise, possibly to be called *information inertia*, gives rise to a cycle type which has not been described before.

V. Cyclical price fluctuations: A transition problem or a case for regulation?

The model in Section II showed the theoretical possibility, not the necessity, of cyclical price fluctuations. So, the fact that we found such price policies in 2002 does not necessarily imply that they are perpetuated and, in fact, they are not. It is possible that the market situation (described by parameters as, for example, α) has changed or that the consumers have learnt about the price policies of some suppliers and now look for suppliers with constant prices. In the latter case, the price fluctuations observed in 2002 might have been merely a temporary problem, rising and fading away during the transition from a monopoly to a truly competitive market. But how can we know?

In our model as well as in reality some firms seem to exploit the incomplete information of many consumers. We cannot exclude that the relatively constant prices in 2004 will be followed by a period of high volatility again. The consumers can be misled by frequent price fluctuations they can only keep track of with prohibitive costs. Though, in our simplified model with price independent aggregate demand, the sum of consumers’ plus producers’ profits is not affected by prices, we suggest that the regulatory authority enforce measures against such policies. One measure is that all suppliers be obliged to announce their price before they deliver their service. Many firms do this already but not all. Even if one decides to go on, i.e. to buy the service though the announced price is higher than last week's price (which one knows), perhaps the next time one will avoid this supplier.

Another measure would be an obligatory period (say four weeks) following a price decrease during which a price can be lowered but not be increased. One must be careful, however, with the introduction of such irreversible price policies. In Bolle and Breitmoser (2004) it is shown, in a general duopoly model, that such restrictions largely facilitate tacit collusion.

So, it is not easy to give clear advise against exploitive cyclical price policies but, nonetheless, this problem should be given attention by the regulatory authority.

Appendix

Proof of Lemma 1: Because of the symmetry of the equilibrium we have to investigate only whether player 1 has an incentive to deviate from D. Let us assume that player 2 chooses s^*_2=(1,r,r,1). Then Player 1's options are as in Figure 9.

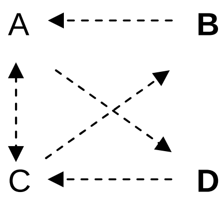

Figure 8: Player 1's options under s^*_2=(1,r,r,1).

The two choices he has in every point (he can stay in the bold letter points) can be composed to cycles D, B, A↔C, ADCA, and ACBA. His profit in B is 0, in A↔C it is (U(A, C) + U(C, B))/2 = 3r/4 which is not larger than the profit in D under (4). In the case of ADCA, Player 1 gets $\frac{1}{6}+\frac{r}{2}$, so that D is again superior under (4). ACDA is always less profitable than D. So, (4) is sufficient for D being an equilibrium.

Are there other strategies s_2 where D prevails as an equilibrium under weaker conditions? For all s_2= (1,.,.,1), again the cycle A↔C is possible. For (.,., 1, 1) the cycle C is possible which leads to the stronger condition r<1/2. s_2= (r, r, r, 1) is excluded by selection principle (i) because D would be isolated. So only s_2= (r, 1, r, 1) remains to be investigated. This strategy provides Player 1 with the options of Figure 9.

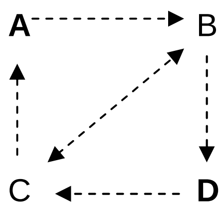

Figure 9: The options of Player 1 under s_2= (r,1,r,1).

In addition to D, cycles A, C $\leftrightarrow$ B, BDCB, ABCA, and ABDCA are possible. ABCA requires r $\leq$ 1/2 to make D an equilibrium, i.e. again a stricter condition than (4) applies.

So, (4) is also necessary for D to be an equilibrium.

Proof of Lemma 2: Let us start with a strategy profile $s_1^* = s_2^* = (1, r, r, r)$ which results in the options visualised in Figure 10.

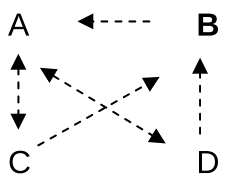

Figure 10: The options of player 1 under $s_2^* = (1, r, r, r)$.

What are Player 1's options, given that Player 2 choose s_2^*? The possible deviations from s_1^* result in cycles B, A $\leftrightarrow$ C, A, C, B, A, and A,,D, B, A. As his profit is 0 for a cycle B, he will never deviate from s_1 in such a way. A $\leftrightarrow$ D is as least as profitable for him as the other cycles if

(13) $$\frac{1}{2}\left(\frac{r}{2}+\frac{1}{2}\right) \geq \frac{1}{2}\left((1+\alpha)\,{}^{r}\!/_{2} + r\left(1-{}^{\alpha}\!/_{2}\right)\right),$$

(14) $$\frac{1}{2}\left(\frac{r}{2}+\frac{1}{2}\right) \geq \frac{1}{3}\left((1+\alpha)\,{}^{r}\!/_{2} + 1-\alpha+\alpha\,{}^{r}\!/_{2}\right),$$

(15) $$\frac{1}{2}\left(\frac{r}{2}+\frac{1}{2}\right) \geq \frac{1}{3}\left(\frac{1}{2} + (1-\alpha)/2 + \alpha\,{}^{r}\!/_{2}\right).$$

These relations are equivalent to (5), (6), and (7) in Lemma 2. Because the respective arguments apply for Player 2 the "if" part of Lemma 2 is proven. In addition, we know that (s_1^*, s_2^*) is not an equilibrium if one of the relations is violated. The question remains whether there are other strategy profiles with the equilibrium cycle A $\leftrightarrow$ D under weaker conditions.

If Player 2 chooses s_2 = (1, r, 1, r) or s_2 = (1, 1, 1, 1) then Player 1 has the opportunity to install the cycle A, C, D, A (and also other cycles). This would always be profitable as

(16) $$\frac{1}{2}\left(\frac{r}{2}+\frac{1}{2}\right) \geq \frac{1}{3}\left((1+\alpha)\,{}^{r}\!/_{2}+1-{}^{\alpha}\!/_{2}+{}^{r}\!/_{2}\right)$$

is equivalent to

(17) $\alpha \geq \dfrac{1+r}{1-r} > 1$ for r > 0 (which is assumed).

If Player 2 chooses s_2 = (1, 1, r, r) then Player 1 can install the cycle B ↔ C as well as A ↔ C. B ↔ C is not more profitable than A ↔ D if α > ½. A ↔ C was also possible under s^*_2 and requires (5). The two requirements (5) and $\alpha \geq$ ½ are more demanding than (5), (6), and (7). (See Figure 4.) So, there are no further parameter values supporting A ↔ D.

Proof of Lemma 3: Let us first regard s_1^* = (1, r, 1, r), s_2^* = (r, 1, r, r). What are Player 1's options, given that Player 2 chooses s_2^*? In addition to B ↔ C, he can install cycles A, B ↔ D, ABDA, and ABCA.

A is always worse for him than B ↔ C. The cycles B ↔ D, ABDA and ABCA are worse under weaker conditions than (14) and (15).

Player 2's options are the same as that of a Player 1 who is confronted with s_2^{**} = (1, 1, r, r). In this case, Player 1 can install A ↔ C, B ↔ D, A ↔ D, ADBCA, and ACBDA. A ↔ C adds the condition (8) and A ↔ D as well as the two four-step cycles add (9).

Thus the "if" part has been proven. It is also clear that the three conditions are necessary for (s_1^*, s_2^*) to be an equilibrium. Is B ↔ C supported by other equilibria?

For s_2 = (r, 1, r, 1) or s_2 = (1, 1, r, 1) Player 1 can install D which is always more profitable than B $\leftrightarrow$ C. s_2 = (1, 1, r, r) has just been investigated. So there are no additional equilibria and Lemma 3 is proven.

Proof of Lemma 4: If Player 2 chooses (r, r, r, r) then Player 1 can only install, in addition to A, the cycles B and A $\leftrightarrow$ B. B is connected with 0 profit while A $\leftrightarrow$ B requires (10) in order not to be more profitable than A.

If Player 2 chooses 1 in B then Player 1 could install the more profitable cycle B $\leftrightarrow$ C. If Player 2 chooses 1 in C or 1 in D then Player 1 would install the cycles C or D which are both more profitable for him than A.

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