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Abstract. This paper continues the analysis of a special uncapacitated single item lot sizing problem where a minimum order quantity restriction, instead of the setup cost, guarantees a certain level of production lots. A detailed analysis of the model and an investigation of the particularities of the cumulative demand structure allowed us to develop a solution algorithm based on the concept of minimal sub-problems. We present an optimal solution to a minimal sub-problem in an explicit form and prove that it serves as a construction block for the optimal solution of the initial problem. The computational tests and the comparison with the published algorithm confirm the efficiency of the solution algorithm developed here.

Keywords: lot sizing problem, minimum order quantity, dynamic programming

1 Introduction

When dealing with industrial projects in the 1970s and 1980s, one of the authors learnt that production managers prefer to determine an optimal production plan by using minimum order quantity (MOQ) restrictions instead of setup cost calculations (see Dempe and Richter, 1981; Richter et al., 1988). Anderson and Cheah (1993) also noticed that in "lot sizing practice out-of-pocket setup costs are commonly accounted for by specifying a minimum batch size parameter" while Constantino (1998) argues that "production below some levels is not allowed, in order to make full use of resources." In practice minor setup costs are often accounted for by providing a minimum order quantity (e.g., Musalem and

Dekker, 2005; Hammond and Raman, 2006). The objective therefore is to minimise only the total inventory costs with respect to the lot size restrictions, and not the sum of setup costs and inventory costs, as in mainstream models. We formulate the single item dynamic lot sizing problem with minimum order quantity restrictions and elaborate on a dynamic programming algorithm for its solution. Performed computational results show that the algorithm is highly efficient and determines an optimal solution in negligible time.

The paper is organised as follows. In Section 2 a short literature review is provided. Section 3 provides the mathematical formulation of the problem with MOQ restrictions in the general case and for the special case without setup costs. The following section introduces the concept of sub-problems and presents the solution algorithm. Section 5 studies minimal sub-problems and elaborates on the bounds of their horizons. In Section 6 we formulate the critical solution of a minimal sub-problem which, as is proven later, is also optimal. Section 7 transfers all developed results onto the so-called unlimited problems, for which end inventories may be positive. The next section checks the efficiency of the developed algorithm on the number of test instances and compares the results achieved with the results of the algorithm developed by Anderson and Cheah (1993). Finally, Section 9 concludes the paper and Appendix contains the proofs of the theorems provided.

2 Literature Overview

Literature on single item lot sizing problems (SILSP) originates from the seminal papers of Wagner and Whitin (1958) and Manne (1958). Since then, intensive research has been carried out on finding efficient solutions and on generalisation models and methods. The results of Federgruen and Tzur (1991) and Wagelmanns et al. (1992) in efficiently solving the classical lot sizing problem (LSP) should be of note. Recent surveys of contributions about LSP can be found in Brahimi et al. (2006) as well as in Drexl and Kimms (1998). Single item lot sizing problems are of practical relevance because they are used as sub-problems in solution processes for the more complex extensions of LSP, like multi-item LSP, see for instance Lasdon and Terjung (1971), Richter (1975, 1982), Maes and Van Wassenhove (1988), Wolsey (1995), Tempelmeier and Derstroff (1996) and others.

To our knowledge, only few papers investigate problems where the minimum order quantity restriction is incorporated into the model. Anderson and Cheah (1993) developed a forward dynamic programming procedure that, for every period, needs a space of states that are defined by the end-period inventory levels. Mercé and Fontan (2003) introduced two heuristics for multi-item single level capacitated lot sizing problems with minimum batch sizes. Furthermore, a dynamic lot sizing model with a stepwise cargo cost function and minimum order amount is analysed in detail by Lee (2004). Porras and Dekker (2006) study the joint replenishment problem for M items with MOQ constrains for each item while Zhou et al. (2007) investigate control policies for stochastic inventory systems with a minimum order quantity. Furthermore, cyclic policies with MOQ restrictions are considered by Kamath and Bhattacharya (2007). Earlier applications of MOQ can be found in Constantino (1998) and Robb and Silver (1998). Besides this, problems with MOQ restrictions can be considered in the context of dynamic lot-sizing models with quantity discounts, where orders less than MOQ are prohibitively expensive.

3 Problem Formulation

Let us consider an uncapacitated single item lot sizing problem (cf. Brahimi et al., 2006):

$$\min \sum_{j=1}^{T} \left(s_j Y_j + p_j X_j + h_j I_j \right) \tag{1}$$

$$I_j = I_{j-1} + X_j - d_j, \tag{2}$$

$$Y_j L_j \leq X_j \leq Y_j d_{jT}, \tag{3}$$

$$Y_j \in \{0, 1\}, \tag{4}$$

$$I_j \geq 0, \quad j = 1, \ldots, T. \tag{5}$$

In model (1)–(5) known parameters T, d_j, s_j, p_j, h_j, and L_j denote the length of the planning horizon, demand values, setup costs, unit production costs, inventory holding costs and the minimum order quantity in periods $j = 1, \ldots, T$, respectively. The decision variables X_j, I_j and Y_j denote the production quantity in period j, the inventory level at the end of period j and the binary variable, which equals unity if production occurs in period j, and zero otherwise.

The sum of production, setup and holding costs is minimised by the objective function (1). The inventory balance equations are provided by (2). Restriction (3) models the fact, that the produced quantity in period j is either zero or at least L_j, where d_{jT} denotes the cumulative demand in periods from j to T. Restriction (4) is obvious, and restriction (5) states that no negative inventories are allowed. Without loss of generality we assume that $I_0 = 0$. It follows from restrictions (3)–(4) that the production quantities are non-negative, hence no special restriction for them is needed.

We set the production costs to zero, because they can be ignored if the whole demand needs to be satisfied (Zhou et al., 2007). In this case model (1)–(5) is reduced to the problem studied in Anderson and Cheah (1993), which appears as a sub-problem in an approximate algorithm to solve a multi-item capacitated lot sizing problem. Furthermore, if MOQ and holding costs are constants and $I_T = 0$, the model appears as a sub-problem to solve a more general remanufacturing lot sizing problem studied by Richter and Gobsch (2005):

$$\begin{aligned}
&\min \sum_{j=1}^{T} I_j \\
&I_j = I_{j-1} + X_j - d_j, \\
&Y_j L \leq X_j \leq Y_j d_{jT}, \\
&Y_j \in \{0, 1\}, \\
&I_j \geq 0, \quad I_0 = I_T = 0, \quad j = 1, \ldots, T.
\end{aligned} \tag{6}$$

We assume that the demand quantities and MOQ are integers. The generalised zero-inventory property for the problem similar to (1)–(5) with the minimum batch restriction was proven by Anderson and Cheah (1993):

Theorem 1 (Anderson and Cheah, 1993). There exists an optimal solution in which

a) $I_{t-1}X_t(X_j - L) = 0$ for each j and t satisfying $1 \leq j < t \leq T$, where $X_j \geq L$ and $X_i = 0$ for each $j < i < t$.

b) If $X_j > L$, then $X_j = d_{jt} - I_{j-1}$ for some $j \leq t \leq T$.

c) $I_T < L$.

Statement a) of the theorem gives a generalised zero-inventory property according to which only the second one of the two subsequent production values with positive inventories between them can be greater than the lower bound L. Similarly to the classic case, statement b) says that the sum of a production value that is greater than the lower bound and the inventory before that period will cover the cumulative demand for some subsequent periods. Finally, statement c) states that the final inventory is always lower than L and therefore is relevant only for problems with a positive final inventory (see Section 7).

We present an efficient dynamic algorithm for problem (6) which, in contrast to the solution given by Anderson and Cheah, does not regard inventory values I_j as states but uses a networked presentation of the problem solving procedure. Emphasis is put on eliminating as many arcs as possible. To our knowledge, algorithms published, so far, for

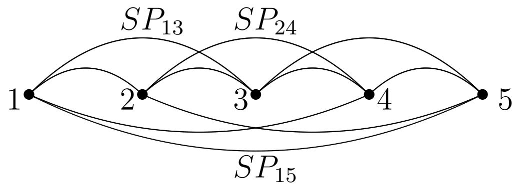

Figure 1: Network presentation of the problem

various lot sizing problems do not draw much attention to the structure of demand inputs. This paper presents an attempt to reduce the complexity of the solution algorithm by considering the special characteristics of demand values, such as the relation of cumulative demand to MOQ and jumps in the demand in various periods.

4 Sub-Problems

In this section we introduce the concept of sub-problems that are created from model (6) with altered start and end periods.

Definition. A *sub-problem* SP_{it} is a part of problem (6) on periods i, $i+1, \ldots, t$ with $I_{i-1} = I_t = 0$, where $1 \leq i \leq t \leq T$.

If the model is presented as a shortest path problem on a graph with T vertices and a maximum of $\frac{1}{2}T(T-1)$ edges, then the value of the objective function of a sub-problem SP_{it} represents the weight of the edge $(i,\, t)$, as demonstrated in Figure 1. To solve a graph problem such as this, the following forward algorithm of Florian and Klein (1971) can be applied. It rests upon the fact that the optimal solution between two nearest generation periods, i.e. between two periods with zero inventories, has special properties, which make the solution more efficient.

$$\begin{aligned}
&\textit{Step 1: Initialisation } i := 1,\ t := 1;\ F_0 := 0,\ F_j := +\infty,\ j = 1, \ldots, T. \\
&\textit{Step 2: } \text{If } F_{i-1} + \hat{I}_{it} \leq F_t \text{ then } F_t := F_{i-1} + \hat{I}_{it} \text{ and } i(t) := i-1. \\
&\textit{Step 3: } \text{If } t < T \text{ then } t := t+1 \text{ and } \textit{return to Step 2}. \\
&\textit{Step 4: } \text{If } i = T \text{ then } \textit{Stop}; \text{ else } i := i+1,\ t := i \text{ and } \textit{return to Step 2}.
\end{aligned} \tag{7}$$

In algorithm (7) variables $\hat{I}_{it}$ stand for the minimal total inventory for the sub-problems SP_{it} and parameters $i(t)$ represent the regeneration periods for disseminating the problem into sub-problems. Furthermore, values F_t denote the minimal cumulative inventory in period t, while F_T contains the objective function value. The optimal production plan

$\hat{X}_1$, $\hat{X}_2 \ldots, \hat{X}_T$ can be determined with the help of regeneration periods $i(t)$ by applying the backward calculation. The time complexity of algorithm (7) is $O(T^2)$.

A sub-problem is *solvable* if $d_{it} \geq L$, while the production quantity must be at least L units and no final inventory is allowed. Formally, the parameter

$$t_i^- = \min\left\{j \geq i \mid d_{ij} \geq L\right\} \tag{8}$$

provides the lower bound for the horizon of the sub-problem SP_{it}. Therefore, sub-problems with $t < t_i^-$ can be excluded from Step 2 of algorithm (7) because they are unsolvable.

5 Minimal Sub-Problems

Not all sub-problems have to be considered in order to solve problem (6). We focus only on one type of sub-problem, called a minimal sub-problem, which represents stable units when constructing an optimal solution for problem (6).

Definition. Sub-problem SP_{it} is *minimal* if there is no such period k, $i \leq k < t$ that $\hat{I}_{it} = \hat{I}_{ik} + \hat{I}_{k+1,\,t}$ and $\hat{I}_k = 0$.

In other words, whatever optimal solution of a minimal sub-problem is found, the inventories for all periods, except the last one, are positive. A sub-problem is not minimal if such period k exists.

Corollary of Theorem 1. For a minimal sub-problem

a) at most one production value is greater than L;

b) this is the last production period for this sub-problem;

c) all other production quantities equal either 0 or L.

The concept of minimal sub-problems is very important for the development of a solution procedure because for dynamic programming only problems such as these should be taken into consideration. They constitute robust elements that cannot be disassembled and are included into an optimal solution as a single block. Hence, leaving aside all non-minimal sub-problems, we reduce the complexity of the solution algorithm. To be able to split the problem into a sequence of minimal sub-problems, we explore the upper bound of the horizon of a minimal sub-problem. For this reason we consider the special characteristics of demand values that indicate the approximate ending of a minimal sub-problem. In the following, we introduce two *critical periods* which are connected with the special properties of the cumulative demand structure.

First, let us consider a period in which the cumulative demand equals a multiple of minimum order quantity L. This period can potentially be the last one for the minimal sub-problem as its end-period inventory equals zero.

Definition. First period j^I, $i \leq j^I \leq t$ of a sub-problem SP_{it} is called the *critical period of the first type* if $d_{i,j^I} = m \cdot L$, where $m \in \mathbb{N}$.

Next, we explore the structure of the cumulative demand and pay special attention to big jumps. The integers $k_j = \lfloor d_{ij}/L \rfloor$, $i \leq j \leq t$, where $k_{i-1} = 0$, allow the determination of the smallest number of minimal lots which suffice to satisfy the cumulative demand d_{ij}. The number k_t is the minimal number of lots which satisfy the total demand of the sub-problem, where all lots except the last one are of size L. If demand is satisfied in every period, then inequalities

$$d_{i,\,J-1} \leq (k_t - 1)L < d_{i,\,J} \tag{9}$$

define the last production period J for the sub-problem SP_{it}.

Definition. First period j^{II}, $i \leq j^{II} \leq t$ of a sub-problem SP_{it} is called the *critical period of the second type* if $k_{j^{II}} - k_{j^{II}-1} > 1$ holds.

This period is critical while it is the first period when the production value must be greater than L, since the cumulative demand $d_{i,\,j^{II}}$ cannot be satisfied by producing only minimal lots. According to Corollary, this should be the last production period of the minimal sub-problem. The next theorem provides the relationships between the critical periods and the horizon of a minimal sub-problem.

Theorem 2 (critical periods). Let SP_{it} be a minimal sub-problem. Then

a) if j^I exists, then $j^I \geq J$ and $d_{j^I+1,\,t} < L$ holds;

b) if j^{II} exists, then $j^{II} = J$ and $d_{j^{II}+1,\,t} < L$ holds.

It follows from Theorem 2 that sub-problems with $d_{j+1,\,t} \geq L$, where $j = j^I,\, j^{II}$, are not minimal and there is no need to analyse them in algorithm (7). Based on two critical periods we can now determine the upper bound t_i^+ for the horizon of the minimal sub-problem SP_{it}:

$$t_i^+ = \min\Big\{ \max\big\{r \mid d_{j^I+1,\,r} < L\big\},\ \max\big\{r \mid d_{j^{II}+1,\,r} < L\big\}\Big\}. \tag{10}$$

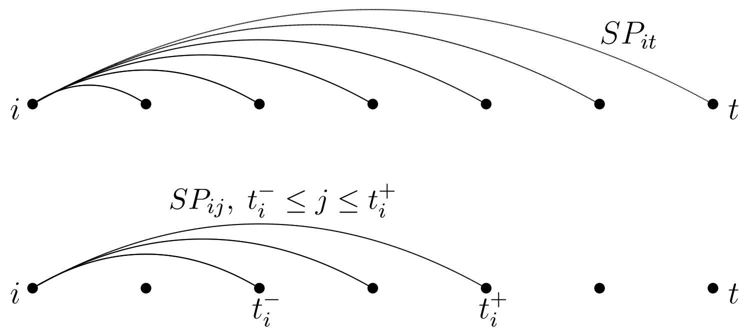

Figure 2: Lower and upper bound for the horizon of a sub-problem

To conclude, for any period i, which constitutes the beginning of a sub-problem SP_{it}, formulas (8) and (10) provide the lower t_i^- and upper t_i^+ bounds of the horizon of the minimal sub-problem. Hence, in the solution algorithm (7) it is unnecessary to consider sub-problems with the horizon smaller or larger than these bounds, as they do not belong to any optimal solution. By disregarding these edges we reduce the complexity of the algorithm, as is illustrated in Figure 2.

6 Solution Algorithm

The solution procedure of problem (6) looks as follows. First of all, we construct the solution of a minimal sub-problem and prove that it is optimal. Next, we provide the algorithm that splits the problem with the horizon T into a series of minimal sub-problems. Finally, we prove that the solution to problem (6) assembled from optimal solutions of its minimal sub-problems is also optimal.

So, let us construct the following solution for a minimal sub-problem SP_{it} with the last production period J:

$$
\begin{aligned}
\tilde{I}_{i-1} &:= 0, && \\
\tilde{I}_j &:= \begin{cases} (k_j+1)L - d_{ij}, & \lfloor d_{ij}/L \rfloor \notin \mathbb{N}, \\ k_j L - d_{ij}, & \lfloor d_{ij}/L \rfloor \in \mathbb{N}, \end{cases} && j = i, \ldots, J-1, \\
\tilde{X}_j &:= \tilde{I}_j + d_j - \tilde{I}_{j-1}, && j = i, \ldots, J-1, \\
\tilde{X}_J &:= d_{Jt} - \tilde{I}_{J-1}, && \\
\tilde{X}_j &:= 0, && j = J+1, \ldots, t, \\
\tilde{I}_{j-1} &:= d_{jt}, && j = J+1, \ldots, t, \\
\tilde{I}_t &:= 0.
\end{aligned}
\tag{11}
$$

We will call solution (11) the *critical solution.*

Theorem 3 (critical solution). The critical solution of a minimal sub-problem is optimal.

Note that it follows from Theorem 3 that if a sub-problem SP_{it} is not minimal, then there always is a number of minimal sub-problems SP_{i,i_1}, $SP_{i_1+1,i_2}, \ldots, SP_{i_k+1,t}$ such that the following relation holds

$$\tilde{I}_{it} = \tilde{I}_{i,i_1} + \tilde{I}_{i_1+1,i_2} + \ldots + \tilde{I}_{i_k+1,t}. \tag{12}$$

Now we improve the dynamic programming algorithm (7) in such a way, that it not only solves problem (6) but also splits it into a series of minimal sub-problems. Additionally, we reduce the complexity of the algorithm by considering only minimal sub-problems instead of revising the cumulative inventories for every value of i and t. The established bounds t_i^- and t_i^+ for the sub-problem's horizon are used to effectively limit the number of sub-problems that come into consideration.

$$\begin{aligned}
&\textit{Step 1: Initialisation } i := 1,\ t := t_1^-;\ \ F_0 := 0,\ F_j := +\infty,\ j = 1, \ldots, T.\\
&\textit{Step 2: } \text{If } F_{i-1} + \tilde{I}_{it} \le F_t \text{ then } F_t := F_{i-1} + \tilde{I}_{it} \text{ and } i(t) := i-1.\\
&\textit{Step 3: } \text{If } t < t_i^+ \text{ then}\\
&\qquad \text{(a) } t := t+1;\ \text{(b) if } t \ge i_{max} \text{ then } t := T;\ \text{(c) } \textit{return to Step 2}.\\
&\textit{Step 4: } \text{If } i = i_{max} \text{ then } \textit{Stop};\\
&\qquad \text{else (a) } i := \max\{i,\ t_1^-\} + 1;\ \text{(b) } t := t_i^-;\\
&\qquad\quad \text{(c) if } t \ge i_{max} \text{ then } t := T;\ \text{(d) } \textit{return to Step 2}.
\end{aligned} \tag{13}$$

In algorithm (13), parameter $i_{max} = \max\{i \ge 1 : d_{iT} \ge L\}$ provides the upper bound for the beginning of the last sub-problem. The complexity of the algorithm is reduced to $O(a \cdot T)$, where $a = \max_i\{t_i^+ - t_i^-\}$. Algorithm (13) will be referred to as *disseminating algorithm.*

Theorem 4 (optimal solution). The disseminating algorithm (13) generates an optimal solution for problem (6) as a series of optimal solutions of minimal sub-problems.

Performance of algorithm (13) is illustrated by the example presented in Table 1 with $T = 7$ periods. Here the MOQ is set to $L = 7$. The results show that apart from the Wagner–Whitin case of an uncapacitated SILSP, the minimal total inventories F_t for the lot sizing problem with minimum order quantities do not increase monotonously. Note that for the problem with seven periods we need only nine iterations of the disseminating algorithm in order to find an optimal solution.

t	t_i^-	t_i^+	1	2	3	4	5	6	7
d_t			5	8	1	4	6	11	6
F_t			∞	∞	∞	∞	∞	∞	∞
$i=1$	*2*	*4*	∞	8	3	11	∞	∞	∞
$i=3$	*5*	*7*	∞	–	–	–	$8+16$	∞	$8+17=25$
$i=4$	*5*	*7*	∞	–	–	–	$3+6$	∞	$3+13=16$
$i=5$	*6*	*7*	∞	–	–	–	–	∞	$11+7=18$
$i=6$	*6*	*7*	∞	–	–	–	–	∞	$9+6=15$
F_t			∞	8	3	11	9	∞	15
X_t			7	7	0	10	0	17	0
I_t			2	1	0	6	0	6	0

Table 1: Illustration of the disseminating algorithm by the example

7 Unlimited Problem

In this section we concentrate on the extension of the model in which the final inventory may be positive, i.e. the restriction $I_T = 0$ has to be replaced with the following constrain

$$I_T \geq 0. \tag{14}$$

We will call problem (6) with restriction (14) the *unlimited* problem. In contrast, problem (6) with $I_T = 0$ considered so far will be referred to as the *limited* problem. Unlimited sub-problems with a positive end-period inventory may be introduced by analogy with limited ones by dropping off the restriction $I_t = 0$. Only the last sub-problem of an unlimited problem may be unlimited.

For an unlimited sub-problem an optimal solution may be either (i) *limited*, with $X_J \geq L$ and $I_T = 0$ or (ii) *unlimited*, with all production quantities not greater than MOQ and $I_T > 0$. If j^{II} exists, then the unlimited solution is not optimal. This is easy to see, since both $d_{j^{II}}$ and $I_{j^{II}-1}$ are greater than L and production quantities from periods before j^{II} can be shifted ahead. Hence, unlimited sub-problems for which a critical period of the second type exists have limited optimal solutions and can be treated as limited ones. Unlimited solutions are worth analysing only if there is no relevant period j^{II}.

Note that there is no lower bound for the horizon of an unlimited sub-problem, thus it is always solvable and should always be considered. In general, the solution of the unlimited problem is always no worse than the solution of the corresponding limited one, since one restriction, i.e. $I_T = 0$, is omitted in the unlimited case. If we introduce the coefficients c_j such that $c_j = 1,\ j < T$ and $c_T = \infty$, and change the objective function to $\sum_{j=1}^{T} c_j I_j \to \min$, we can transform an unlimited problem into a limited one.

Now we can construct an unlimited critical solution:

$$
\begin{aligned}
\tilde{I}_{i-1} &:= 0, \\
\tilde{I}_j &:= \begin{cases} (k_j+1)L - d_{ij}, & \lfloor d_{ij}/L \rfloor \notin \mathbb{N}, \\ k_j L - d_{ij}, & \lfloor d_{ij}/L \rfloor \in \mathbb{N}, \end{cases} && j = i, \dots, t, \\
\tilde{X}_j &:= \tilde{I}_j + d_j - \tilde{I}_{j-1}, && j = i, \dots, t.
\end{aligned} \tag{15}
$$

Since j^{II} does not exist, the unlimited critical solution can be easily proved to be feasible and optimal, provided the limited critical solution is no better.

8 Computational Study

To prove the efficiency of the developed disseminating algorithm we conducted an extensive computational study. We tested the algorithm on the number of randomly generated instances and compared its performance with the results of the forward procedure of Anderson and Cheah (1993). For computational experiments we approximated the demand values with normal distribution with three different mean values: small mean ($\mu = 40$), middle ($\mu = 200$) and large ($\mu = 600$). Furthermore, for every mean level we selected two values of variance that corresponded to small and large demand fluctuations. In total we received six groups of problems.

Similarly to the experiment by Anderson and Cheah, for every problem group, we set three levels of minimum order quantity. The demand values that are smaller than MOQ are called *low volume* (LV) values and demand values greater than MOQ are referred to as *high volume* (HV) values. In this context we selected the minimal lot size values so that the low volume to high volume ratio for the demand equals 1 : 3, 1 : 1, and 3 : 1 (cf. Anderson and Cheah, 1993). In other words, if LV : HV = 1 : 3, this means that 25 per cent of demand values in a sample are less than MOQ and 75 per cent are greater.

The results of the computational study for $T = 50$ periods are presented in Table 2. The column *Number of iterations* indicates how many times *Step 2* of the algorithm (13) has to be performed, i.e. how many times we should compute $\tilde{I}_{it}$ and compare $F_{i-1} + \tilde{I}_{it}$ with F_t in order to find the optimal solution. The similar parameter for the algorithm of Anderson and Cheah is presented in the rightmost column *Number of states*, which are defined by the end-period inventory levels I_t. Both algorithms were coded in Java 1.6 and run on a 2 GHz Intel Core 2 Duo machine with 2 GB memory running Windows Vista. All data in the table are aggregated over ten randomly generated instances.

As can be seen from Table 2, the disseminating algorithm outperforms the forward procedure of Anderson and Cheah in both computational time and in the number of operations needed to find an optimal solution. The computational time of the disseminating algorithm is on average 2,5 times shorter for low MOQ (LV : HV = 1 : 3), six times shorter for MOQ which equals the demand mean and 13,7 times shorter for high values of MOQ (LV : HV = 3 : 1). Even if we can be accused of having not selected the best data structure and procedures for programming the algorithm of Anderson and Cheah, the number of iterations versus the number of states clearly speaks in favour of the disseminating algorithm. Our algorithm gains further authority, when the minimum order quantity is relatively high compared with the demand mean; in this situation there are many low volume items that have to be united in one sub-problem in order to make it solvable. Therefore, the lower bound for the horizon of a sub-problem is relatively high, what eliminates unpromising edges.

9 Conclusions

The paper continues the analysis of a special uncapacitated single item lot sizing problem, where a minimum order quantity restriction, instead of the setup cost, guarantees a certain level of production lots. The detailed analysis of the model and investigation of the particularities of the cumulative demand structure allowed us to develop a solution algorithm based on the concept of minimal sub-problems. Furthermore, we have presented an optimal solution to a minimal sub-problem in an explicit form and have proved that it serves as a construction block for the optimal solution of an initial problem. The empirical test and the comparison with the published algorithm confirm the efficiency of the solution algorithm we have developed.

Appendix

Proof of Theorem 2.

a) If we assume that $d_{j^I+1,t} \geq L$, then SP_{it} can be split into two parts, what contradicts the statement about its minimality. If we further assume that $j^I < J$ then from the inequalities $d_{i,j^I} \leq d_{i,J-1} \leq (k_t-1)L$ it follows that $d_{i,j^I} + L \leq k_t L \leq d_{it}$. This leads to the restriction $d_{j^I+1,t} \geq L$ which contradicts the previous proposition.

Low volume to high volume ratio	Disseminating algorithm		Anderson & Cheah's algorithm	
	Comput. time, ms	Number of iterations	Comput. time, ms	Number of states
$\mu = 40,\ \sigma = 4$				
1 : 3	6,2	1016	15,6	1296
1 : 1	3,7	798	25,0	1505
3 : 1	7,6	813	60,7	2369
$\mu = 40,\ \sigma = 12$				
1 : 3	7,7	977	15,7	1292
1 : 1	4,7	712	20,3	1414
3 : 1	4,7	645	82,8	2676
$\mu = 200,\ \sigma = 20$				
1 : 3	3,2	969	14,0	1304
1 : 1	3,1	735	21,5	1502
3 : 1	4,8	838	52,8	2399
$\mu = 200,\ \sigma = 60$				
1 : 3	9,4	983	15,8	1306
1 : 1	2,2	678	24,9	1547
3 : 1	6,4	627	120,0	3335
$\mu = 600,\ \sigma = 50$				
1 : 3	6,3	942	15,6	1302
1 : 1	7,7	803	18,8	1498
3 : 1	7,8	881	56,2	2373
$\mu = 600,\ \sigma = 150$				
1 : 3	7,7	975	15,5	1302
1 : 1	6,1	724	25,2	1581
3 : 1	6,3	752	124,6	3348

Table 2: Computational results for the disseminating algorithm vs. forward algorithm of Anderson & Cheah (1993)

b) Let as assume that $J < j^{II}$, i.e. we will not produce after period J. Then the sequence of the following inequalities holds $k_t \geq k_{j^{II}} > k_{j^{II}-1} + 1 \geq k_{j^{II}-1} + 2 \geq k_J + 2$ and therefore $d_{iJ} > (k_t - 1)L \geq (k_J + 1)L$ is also true. This contradicts, however, the inequality $d_{iJ} < (k_J + 1)L$ which follows from the definition of k_j.

On the other hand, let us assume that $j^{II} < J$ and thus $X_{j^{II}} \in \{0, L\}$. According to part a) of this Theorem, if j^I exists, then $J \leq j^I$ and $d_{i,\,j^{II}} > k_{j^{II}} L \geq (k_{j^{II}-1}+2)L$ holds. Until period $j^{II} - 1$ we produced $X_{i,\,j^{II}-1} = (k_{j^{II}-1} + 1)L$ units and till period j^{II} we produced at most $X_{i,\,j^{II}} \leq (k_{j^{II}-1}+2)L$. In this case, however, inventories $I_{j^{II}} = X_{i,\,j^{II}} - d_{i,\,j^{II}} < 0$, what leads to contradiction.

So, we have proven that $j^{II} = J$ and therefore $d_{j^{II}+1,\,t} < L$. Otherwise the production quantities could be shifted ahead and the sub-problem would not be minimal. □

Proof of Theorem 3.

First we will prove that the critical solution is feasible. It is obvious, that (11) satisfies the flow restriction from (6). Furthermore, since $\tilde{X}_j = \tilde{I}_j + d_j - \tilde{I}_{j-1}$, the second restriction from (6) holds for $j < J$. From (9) we further have that $k_{J-1}L < d_{i,\,J-1} \leq (k_t - 1)L$. Therefore, $k_{J-1} + 1 \leq k_t - 1$ and $\tilde{X}_J = d_{Jt} - \tilde{I}_{J-1} = d_{Jt} + d_{i,\,J-1} - (k_{J-1} + 1)L \leq d_{it} - k_t L + L \geq L$, what proves that the second restriction also holds for $j = J$. Finally, from Theorem 2a it follows that $k_j L < d_{ij} < (k_j + 1)L$ for $j < J$, what proves the restriction $\tilde{I}_j > 0$. Therefore, the last restriction from (6) is also satisfied.

Next we prove that the critical solution is optimal. Let $\hat{X}_j,\ \hat{I}_j,\ i \leq j \leq t$ and $\hat{J}$ denote the production and inventory values as well as the last production period for an optimal solution of the minimal sub-problem SP_{it}. It is obvious that $\hat{I}_j = \tilde{I}_j$ for $\hat{J} \leq j < t$. Let us assume that there exists such period $j < \hat{J}$ that $\hat{I}_j < \tilde{I}_j$. Then, however, in the optimal solution there is at least one production period less, i.e., $\hat{I}_j \leq k_j L - d_{ij} < 0$ holds and the solution is infeasible. Next, let's $j < \hat{J}$ denote the first production period for which $\hat{I}_j > \tilde{I}_j$ holds. Then in the optimal solution there is at least one production period more, hence $\hat{I}_j \geq (k_j + 2)L - d_{ij}$ and $\hat{I}_{j-1} = (k_{j-1} + 1)L - d_{i,\,j-1}$. Then $\hat{I}_j \geq L$ and $\hat{I}_{j-1} = \hat{I}_j - \hat{X}_j + d_j \geq d_j$. Therefore, production quantity from period j can be shifted ahead and the solution is not optimal. Hence, $\hat{I}_j = \tilde{I}_j$ holds also for $j < \hat{J}$. □

Proof of Theorem 4.

We have to prove that pairs $\big(i(t)+1,\,t\big)$ generate minimal sub-problems. If a sub-problem is not minimal, then the value $\hat{I}_{i(t)+1,\,t}$ can be decomposed as provided in relation (12). According to the algorithm (13), $F_{i(t)} + \tilde{I}_{i(t)+1,\,t} < F_{i_k} + \tilde{I}_{i_k+1,\,t}$ holds. Furthermore, the equality $\hat{I}_{i,\,i_k} = \tilde{I}_{i,\,i_1} + \tilde{I}_{i_1+1,\,i_2} + \cdots + \tilde{I}_{i_{k-1}+1,\,i_k}$ is fulfilled and $F_{i_k} \leq F_{i(t)} + \tilde{I}_{i(t)+1,\,i_k}$ holds. Then, however, the strong relation $\tilde{I}_{i(t)+1,\,t} < \tilde{I}_{i(t)+1,\,i_k} + \tilde{I}_{i_k+1,\,t} = \hat{I}_{i(t)+1,\,t}$ holds, what leads to contradiction. □

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